NATURALNESS AND FINE-TUNING

THEORETICAL AND PHILOSOPHICAL ASPECTS OF THE HIERARCHY PROBLEM

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"If one has to summarise in one word what drove the efforts in physics beyond the Standard Model of the last several decades, the answer is naturalness."

- Gian Francesco Giudice [1]

Head of the Theoretical Physics Department, CERN

OUTLINE

- Definitions of Naturalness and Fine-Tuning
- Illustrative Example: The Infrared-Ultraviolet Connection
- Naturalness Problems of the Standard Model
- Case Study: The Hierarchy Problem
  - What is the Hierarchy Problem?
  - Quantum Corrections and Renormalization
  - Derivation of the Quadratic Divergence of Higgs Mass
  - Higgs Mass Fine-Tuning vs. Other Particles
  - Solutions to the Hierarchy Problem
  - Is the Hierarchy Problem (Really) a Problem?
- The Post-Naturalness Era

THE (MANY) DEFINITIONS OF NATURALNESS

Naturalness is…

…the requirement that the numerical values of a theory's parameters do not rely on fine-tuned cancellations.[2]

…the idea that a relation between two parameters in quantum field theory is "natural" if, due to effects of spontaneous symmetry breaking, it only receives finite radiative corrections. [3]

…a prohibition on delicate sensitivity between physics at different energy scales. [4]

Naturalness is not…

… a criterion of consistency nor a requirement for agreement with experimental data. [2]

WHAT DOES NATURALNESS LOOK LIKE IN PRACTICE?

THE INFRARED-ULTRAVIOLET CONNECTION

- Motivation: Natural theories do not rely on unlikely cancellations or display delicate sensitivity between physics at different energy scales
- Small changes of the fundamental parameters at the UV scale should not lead to drastic changes at the IR scale (SM)[5]
- Problem: Changing the parameters in the UV is not a physical variation.
- Statements about the "unlikely" fine-tuning require a probability distribution of the parameters of our universe, which we don't have

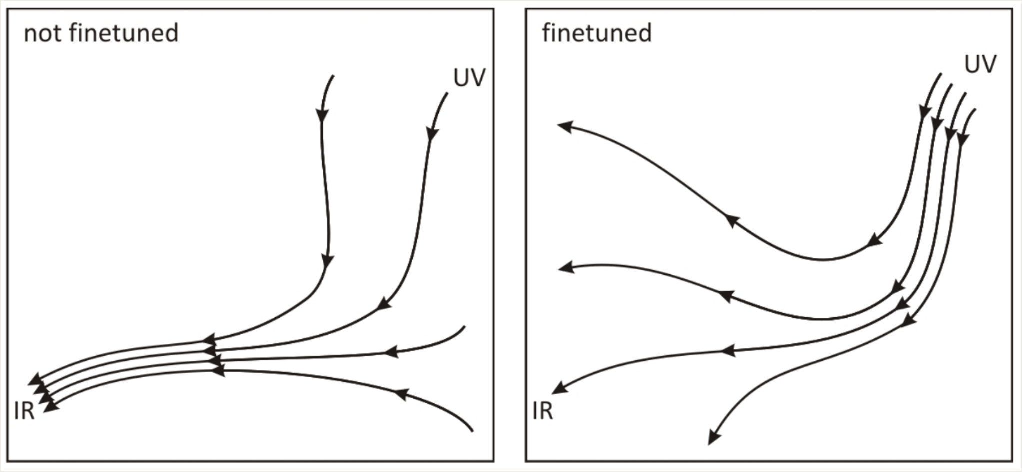

Fig.: Fine-tuning in theory space[6]

- Quantifying fine-tuning is arbitrary by definition, since it relies on a choice of a presumed probability distribution

NATURALNESS PROBLEMS OF THE STANDARD MODEL

- Over the past decades, multiple naturalness problems of the SM have been identified:
- Hierarchy problem
  - Concerns the stability of the Higgs mass due to quantum corrections
- Strong CP problem
  - Involves the QCD parameter which introduces CP violation: $\theta < 10^{-10}$; no explanation why this angle is so small
- Flavor hierarchy problem
  - Unexplained span over several orders of magnitude of the Yukawa couplings
- Additionally: Cosmological constant problem
  - Observations indicate that the cosmological constant is extremely small, about 120 orders of magnitude smaller than the theoretical prediction

CASE STUDY:
THE HIERARCHY PROBLEM

WHAT IS THE HIERARCHY PROBLEM?

- The Higgs boson mass appears unnaturally light when compared to the large quantum corrections.[4]

$$m_H^2 = m_{H,0}^2 + \delta m^2$$
$$\approx m_{H,0}^2 - \frac{3y_t^2}{8\pi}\Lambda^2 + \mathcal{O}\left(m_{weak}^2\right)$$

- $m_H = 125 GeV$ is the measured Higgs mass
- $m_{H,0}$ is the unobservable bare mass
- δm are the quantum corrections to the mass
- y_t is the top quark Yukawa coupling

- Λ is renormalization cutoff, usually interpreted to be the scale of new physics
  - For example: M_{GUT}, M_{Planck} etc.
- The bare Higgs mass $m_{H,0}$ must be fine-tuned to fit the observation:

$$\mathcal{O}\left(10^4\right) = \mathcal{O}\left(10^{38}\right) - \mathcal{O}\left(10^{38}\right) \quad \text{for } \Lambda \approx 10^{19}\ GeV$$

- Worst-case scenario (at Planck scale): required cancellation to one part in 10^{34}
- Additionally, the quadratic dependence is considered too sensitive to variations in Λ

QUANTUM CORRECTIONS AND RENORMALIZATION

$$m_H^2 = m_{H,0}^2 + \delta m^2 \quad \rightarrow \quad \text{What is } \delta m^2 \text{ ?}$$

- Quantum corrections result from particle interactions with virtual particles
  - Loop diagrams often involved, with the number of loops in the Feynman diagram representing higher-order corrections
  - Calculation in form of loop integrals, which often diverge

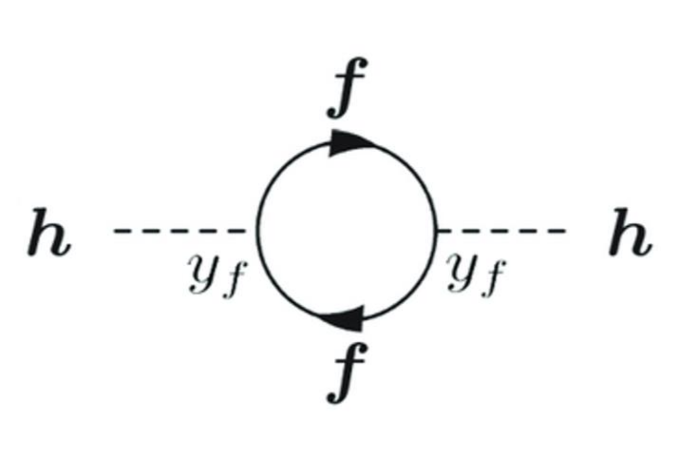

Fig.: Divergent one-loop contribution to the Higgs mass[8]

UV Divergences:[7]

$$Loop\ integral \propto \int \frac{d^4k}{k^{2n_b+n_f-I}}$$

k = loop momentum
n_b = # of boson propagators
n_f = # of fermion propagators
I = powers of k from interactions

Converges when: $2n_b + n_f - I > 4$

Renormalization:

1. Choose regularization scheme to handle the infinities (e.g., cutoff, dimensional…)
2. Redefine the bare parameters in terms of the physical and regularization parameters (e.g., on-shell or $\overline{MS}$ scheme)

QUADRATIC DIVERGENCE OF THE HIGGS MASS

- As an example, the quadratic divergence of the Higgs mass can be derived by calculating the top quark one loop quantum correction:[9]
- Self-energy correction $\Pi_H(p^2)$ for the Higgs field at zero external momentum ($p^2 = 0$):

$$-i\Pi_H(0) = -3\left(\frac{iy_t}{\sqrt{2}}\right)^2 \int \frac{d^4k}{(2\pi)^4} Tr\left[\frac{i(\not{k} + m_t)}{k^2 - m_t^2}\,\frac{i(\not{k} + m_t)}{k^2 - m_t^2}\right]$$

where $\frac{i(\not{k}+m_t)}{k^2-m_t^2}$ is the fermion propagator, $\not{k} = k^\mu \gamma_\mu$ and γ_μ are the gamma matrices. If $k^2 \gg m_t^2$:

$$-i\Pi_H(0) = -3\left(\frac{iy_t}{\sqrt{2}}\right)^2 \int \frac{d^4k}{(2\pi)^4}\; Tr\left[\frac{i\not{k}}{k^2}\,\frac{i\not{k}}{k^2}\right] = -3\,\frac{y_t^2}{2} \int \frac{d^4k}{(2\pi)^4}\,\frac{4k^2}{k^4}$$

- <u>Renormalization:</u> Introduce the momentum cutoff parameter Λ and evaluate the integral:

$$\delta m_{H,t}^2 = Re[-i\Pi_H(0)] = -3\,\frac{4y_t^2}{16\pi^2}\int_0^\Lambda dk\, k = -\frac{3y_t^2}{8\pi^2}\Lambda^2$$

- This result is the mass quantum correction in the case of a top quark loop.

HIGGS MASS FINE-TUNING VS. OTHER PARTICLES

- Excluding light fermions, the leading order corrections to the Higgs mass are:[10]

$$m_H^2 = m_{H,0}^2 + \frac{3}{16\pi^2 v^2}\left(m_H^2 + 2m_W^2 + m_Z^2 - 4m_t^2\right)\Lambda^2$$

- Only the Higgs boson mass diverges quadratically, due to its scalar particle nature

- What happens for fermions and gauge bosons?
  - Fermion and gauge boson mass corrections display a logarithmic divergence with the momentum cutoff parameter Λ:[11]

$$m_f \approx m_{f,0} + \frac{3\alpha}{4\pi} m_{f,0} \log\left(\frac{\Lambda^2}{m_{f,0}^2}\right)$$

  - This is due to protective symmetries, which constrain the forms the corrections can take, leading to less severe divergences

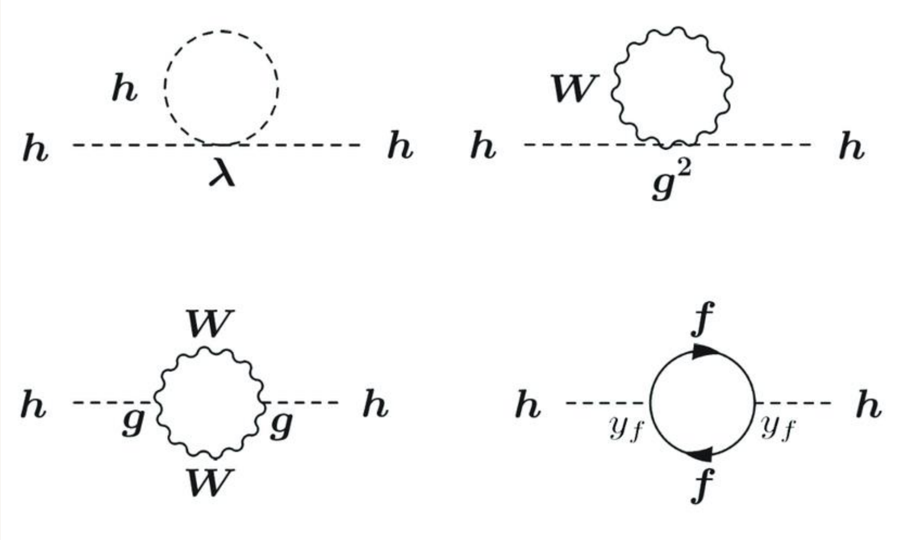

Fig.: Feynman diagrams of the one-loop corrections to the Higgs mass[8]

SOLUTIONS TO THE HIERARCHY PROBLEM

- The fine-tuning of the Higgs mass has served as a strong motivator in the development of many BSM theories proposed to solve the hierarchy problem
- These include (but are not limited to):[1]
  - Supersymmetry: Contributions of supersymmetric particles to the quantum corrections cancel out those of the SM particles, leading to a logarithmic divergence
  - Extra dimensions: Theories that solve the hierarchy problem by proposing extra spatial dimensions which effectively reduce the Planck scale
  - Composite Higgs: The Higgs is a composite particle made up of more fundamental constituents, hence is does not have the quadratic divergence of scalar particles
- So far, none of these theories have been experimentally confirmed

IS THE HIERARCHY PROBLEM (REALLY) A PROBLEM?

- The answer to this question hinges upon the interpretation of Λ[12]
- Recently, many have pointed out that the hierarchy problem is both regularization and renormalization scheme dependent:
  - Cutoff regularization and $\overline{MS}$ renormalization are necessary
  - Physical predictions, however, should not depend on these choices
  - Conclusion: The hierarchy problem is a mere artifact of mathematical convention
- Others argue that this dependency is not a problem, if Λ is interpreted as a physical parameter of the scale at which the SM breaks down:
  - In this case, the dependency reflects a preferred parametrization on the universe

SUMMARY: THE HIERARCHY PROBLEM

- The Higgs mass quantum corrections diverge quadratically with the cutoff parameter:

$$m_H^2 = m_{H,0}^2 + \frac{3}{16\pi^2 v^2}\left(m_H^2 + 2m_W^2 + m_Z^2 - 4m_t^2\right)\Lambda^2$$

- This necessitates fine-tuning in the order of 10^{34} to recover the observed Higgs mass
- The divergence of corrections arises from calculations of loop integrals via renormalization
- For other particles, these calculations yield merely logarithmic divergences due to protective symmetries
- Of the theories proposed to solve the problem of the Higgs mass naturalness, none have been experimentally confirmed
- This has led to a closer examination of the Hierarchy problem and the underlying naturalness principle

THE POST-NATURALNESS ERA

- Naturalness has had modest empirical success: Discovery of the charm quark in 1974[13]
- It was strongly believed that BSM physics should be found at the energies probed by the LHC
  - In the past decade, the absence of any such signals is ruling out more and more of the parameter spaces of these theories
- Retrospectively, many have criticised various aspects of naturalness and its use:[14]
  - Naturalness is an extra-empirical criterion, as such it should be used with caution
  - They argue that at times the principle has been bent to fit observations rather than using observations to test the principle[3]
- Nonetheless, some others still praise naturalness as a valid hypothesis[15] and as of right now there is no consensus on its status within particle physics

CONCLUSION

- Despite the prevalent notion of physics as an exact and objective discipline, it at multiple points contains philosophical and metaphysical underpinnings
- When these go unexamined, they can lead to misconceptions and hinder scientific progress
- It is not enough to discard of the naturalness principle, we have to understand what went wrong and why, to avoid repeating the mistakes
- If naturalness is indeed a false requirement for the SM, a paradigm shift might be needed in the development of future theories
- Other aspects of particle physics that lend themselves to philosophical ponderings include:
  - The ontological implications of virtual particles
  - The prevalence of the multiverse hypothesis in current theories
  - The impact of computer simulations on the epistemic status of LHC data

SOURCES

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